

Missing Learning Opportunities in Classroom Instruction: Evidence from an Analysis of a Well-Structured Lesson on Comparing Fractions

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Abstract: This paper analyzed a well-structured lesson to examine the opportunities and missing opportunities for students' learning in terms of developing basic skills and higher-order thinking skills. The instructional activities in the lesson that show a very systematic choice of variation and clear focus, may serve well the goal of teaching a specific basic skill. However, the type of engagement the teacher created in the lesson is less ideal for fostering students' higher-order thinking skills. While both basic and higher-order thinking skills are important and it is not necessary to sacrifice basic skills for higher-order thinking, nor higher-order thinking for basic skills, this paper calls for research and design classroom instruction to develop both basic skills and higher-order thinking.

Key words: Higher-order thinking skills; Basic skills; Fractions; Mathematics learning and teaching

Introduction

Although recent research in cognition has shown that students' experiences out of school have a substantial effect on their learning and problem solving (Lave, 1988; Resnick, 1987a), classroom instruction is still considered a central component for understanding the dynamic processes and the organization of students' thinking and learning (Cai, 2004; Rogoff & Chavajay, 1995; Wozniak & Fischer 1993). Bruner (1990) proposed that it is culture and education, not biology, which shapes human life and the human mind. Because classroom instruction plays such a central role in students' learning, researchers have long tried to characterize the nature of the

classroom instruction that maximizes students' learning opportunities (Brophy & Good, 1996; National Academy of Education, 1999). The purpose of this paper is to discuss students' learning opportunities and missing opportunities in a well-structured lesson.

When we discuss the nature of mathematics instruction, what we first encounter is determining the criteria we should use to make the judgment. In this paper, we take a position that the quality of mathematics instruction can only be judged by two criteria: by desirable outcomes in students' learning and the processes that yield those desirable learning outcomes (Cai, 2004). This position has strong support from the current views of mathematics learning and research on teaching and teacher development (e.g., Bransford, Brown, & Cocking 2000; Cobb, 1994; Hatano, 1993; Schoenfeld, 2000; Shulman, 1987; Steffe, Nesher, Cobb, Goldin, & Greer, 1996; Wang, Haertel, & Walberg, 1993). Therefore, discussing the nature of classroom instruction requires a careful analysis of what mathematics students do learn in classroom. It also requires a careful analysis of the classroom events that are instructionally guiding students' learning of important mathematics.

For the desirable learning outcomes, we take a position that mathematics instruction should foster students' development of both basic skills and higher-order thinking skills. While there are no commonly accepted definitions of higher-order thinking skills in mathematics, researchers and educators generally agreed that higher-order thinking skills involve the abilities to think flexibly to make sound decisions in complex and uncertain problem situations (Resnick, 1987b). In addition, higher-order thinking skills involve self-monitoring one's own thinking—metacognitive skills. In particular, mathematics instruction should provide students with opportunities to: (a) think about things from different points of view; (b) step back to look at things again; and (c) consciously think about what they are doing and why they are doing it.

While we all agree that in mathematics it is important for students to have basic algorithmic knowledge and computation skills to solve many kinds of problems, this does not ensure that they have the conceptual knowledge and higher-order thinking skills to solve non-routine or novel problems (e.g., Cai, 2001; Hatano, 1993; Steen, 1999; Sternberg, 1999). For example, in one of a series of studies examining U.S. and Chinese 6th grade students' mathematical problem solving and problem posing (Cai, 2001), it was found that the Chinese sample had significantly higher mean scores than did the U.S. sample on the process-constrained tasks. However, the U.S. sample had significantly higher mean scores than did the Chinese sample on the process-open tasks. Process-constrained tasks refer to problems that can be solved by executing a "standard algorithm." In contrast, process-open tasks are problems that usually cannot be solved by an algorithm, and more typically require novel

exploration of the problem situation. Furthermore, a process-open task usually lends itself to a variety of acceptable solutions.

The above results clearly show the complexity of characterizing the relationships between developing basic skills and conceptual understanding and higher order thinking skills in mathematics. Although some researchers believe that there should not be a tension between developing basic skills and higher-order thinking skills, in reality, teachers face challenges to develop both basic and higher-order thinking skills in classroom.

In analyzing the classroom events, we will focus on the following two critical aspects of the classroom instruction: Mathematical tasks and classroom discourse (Cai, 2003; Cazden, 1986; Hiebert & Wearne, 1993). It is recommended that in classrooms students should be exposed to truly problematic tasks so that mathematical sense-making is practiced (NCTM, 1991, 2000). Mathematical tasks govern not only students' attention to particular aspects of content, but also their ways of processing information (Doyle, 1983; Doyle, 1988; Hiebert & Wearne, 1993; NCTM, 1991, 2000; Stein, Grover, & Henningsen, 1996). However, only "worthwhile problems" give students the chance to solidify and extend what they know and stimulate mathematics learning (NCTM, 1991). Regardless of the context, worthwhile tasks should be intriguing, with a level of challenge that invites speculation and hard work. Most importantly, worthwhile mathematical tasks should direct students to investigate important mathematical ideas and ways of thinking toward the learning goals. Doyle argues that tasks with different cognitive demands are likely to induce different kinds of learning. Mathematical tasks, which are truly problematic, have the potential to provide the intellectual contexts for students' mathematical development (Doyle, 1988). Such tasks can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity (NCTM, 1991).

Worthwhile mathematical tasks alone do not guarantee students' learning. They are important, but not sufficient, for effective mathematics instruction because worthwhile tasks may not be implemented as intended. Stein et al. found that only about 50% of the tasks that were set up to require students to apply procedures with meaningful connections were implemented effectively (Stein, Grover, & Henningsen, 1996). In the classroom, students' actual opportunities to learn depend not only on the type of mathematical tasks that teachers present, but also on the kind of discourse that teachers orchestrate to implement the tasks toward learning goals. (Cazden, 1986).

In this paper, we examined the challenges a teacher face to develop both basic skills and higher order thinking skills. In addition, we used the two constructs (mathematical tasks and classroom discourse) to examine the opportunities and missing opportunities in fostering students' development of both basic and higher order thinking skills in the lesson.

Background

The lesson was taught by Miss Chen, who has been working closely with the officials from the Curriculum Development Institute in Hong Kong. She is a very motivated teacher and has a desire of making changes in practice following the recommendations of curriculum reform in Hong Kong. Besides aiming to develop the basic skills in a sequence of very well structured examples and activities, the teacher tried very hard to provide students with opportunities for communication and mathematical reasoning. The students in her class are quite typical in terms of their family background or achievement level in Hong Kong. It is a fourth grade classroom. The lesson focused on comparing fractions. Before this lesson, students had learned the meaning of fractions, equivalent fractions and comparing proper fractions (fractions less than 1) with the same denominators, same numerators or different denominators and numerators.

The teacher was interviewed prior and after the lesson to understand her planned and implemented actions. Two researchers from the Hong Kong Curriculum Development Institute have observed the lesson. The lesson was carried out in Cantonese and video-recorded. The excerpts included in the subsequent analysis are translated from the verbatim transcript. The interviews, observers' notes and the video-taped lesson with transcripts were used in our analysis of the lesson.

Analysis of the Lesson

Applying an algorithm: comparing $2\frac{1}{2}$ and $1\frac{1}{2}$; $2\frac{3}{5}$ and $2\frac{1}{5}$

The teacher first showed a picture in which two and a half circles were shaded and the teacher asked the students to represent the picture with a fraction. Based on the visual clues, the students gave the expected answer easily and one student was asked to write the larger than symbol ">" between $2\frac{1}{2}$ and $1\frac{1}{2}$. Then the teacher asked the question why. A student gave the answer that the integer part was greater. This answer was well supported by the teacher and she further pointed out that the fractional parts were the same and need not to be compared.

EXCERPT 1

S4: Because the integer is bigger than the other integer.

T: Correct. The integer is bigger than the other integer. Should we neglect the fractional parts after comparing the integer parts of two fractions? No, we need to compare the fractions. How are the fractions? Raise your hands.

T: S5.

S5: The same.

T: The same. Correct. Then we do not need to care for the fractions and we only compare the whole numbers for this pair.

For the next example, the teacher asked the students to compare $2\frac{3}{5}$ and $2\frac{1}{5}$ without showing any pictures. A student gave the right answer for which many students agreed. In probing her class for how they got the answer, they agreed on an algorithm in comparing mixed fractions.

EXCERPT 2

T: Many students agreed with this. Why is this good? S7, what method was used to compare? First, what do we compare?

S7: Numerator.

T: What should we see first if we compare mixed fractions?

S7: The integers.

T: Yes. We first compare the integers, what next?

S7: Compare the fractions.

T: What?

S7: The denominators.

T: The denominators. Well. Thank you for trying. Do we remember what we should look for when we compare the fractions? (The teacher faced the class.) S8.

S8: The numerators.

T: The ...

S8: The denominators.

T: The denominators. First see if the denominators are the same, shouldn't we?

All: Yes.

T: Good. Then we now compare the integers. Because they are the same, then we see the denominators. Is one fifth or three fifth larger? Do you remember when I cut the cake into 5 parts. Which should be larger, three or one?

S9: Three parts.

(The teacher wrote “whole numbers → fractions →same denominators→” on the board during this segment of discussion.)

This segment of the lesson, which lasted about five minutes, had the purpose of preparing the students to differentiate the integer and the fractional parts in the process comparing two mixed fractions. The first example ($2\frac{1}{2}$ and $1\frac{1}{2}$) and the second example ($2\frac{3}{5}$ and $2\frac{1}{5}$) deliberately keep either the integer or the fractional parts constant, leading the students to focus and compare a part of the mixed fraction only. The first example guided the students to compare the integer parts of the fractions and the second example guided the students to compare the fractional parts of the fractions. Furthermore, the teacher’s style of questions belongs to a style which led the class to follow a specific path closely. The students were led to inspect different parts of the fractions in a specific sequence (whole numbers, fractions, denominators) and they were not encouraged to skip any part.

A challenge: comparing $3\frac{2}{3}$ and $3\frac{3}{4}$

The teacher then told the class that they were ready for a challenge to compare $3\frac{2}{3}$ and $3\frac{3}{4}$. She asked the students to discuss among themselves for about one minute.

After the teacher had resumed the attention of the whole class, the class suggested to make the denominators bigger or to get the common denominators. This was obviously matching the teacher’s expectation. The teacher then completed her listing of the steps in the algorithm “whole numbers, → fractions →same denominators, → making denominators larger, → getting common denominators” on the board and she showed a form with some blanks for the students to fill in (figure 1). After completing the forms, the teacher asked the students to compare $3\frac{8}{12}$ and $3\frac{9}{12}$ and tell the characteristics before they started to compare the two fractions. That is, the two fractions had the same denominators and integers. After this, the teacher showed the pictures so that they were reassured by the answers.

EXCERPT 3

- T: Good. After making the denominators bigger, what are the characteristics, S13? First, what are the same?
 S13: The denominators are the same.
 T: There is something else.

S13: The integers are the same.
 T: Now the denominators and the integers are the same. Can we do the comparison?
 Class: Yes.

$3\frac{2}{3}$ $= () \frac{2 \times ()}{3 \times ()}$ $= () \frac{()}{()}$	$3\frac{3}{4}$ $= () \frac{3 \times ()}{4 \times ()}$ $= () \frac{()}{()}$	After the students filled in the blanks, it became this. \longrightarrow	$3\frac{2}{3}$ $= (3) \frac{2 \times (4)}{3 \times (4)}$ $= (3) \frac{(8)}{(12)}$	$<$ $<$	$3\frac{3}{4}$ $= (3) \frac{3 \times (3)}{4 \times (3)}$ $= (3) \frac{(9)}{(12)}$
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Figure 1. Teacher’s presentation structure for comparing $3\frac{2}{3}$ and $3\frac{3}{4}$

Recapitulation of the algorithm

The method became more explicit in the class discourse of the subsequent activities. In the next activity, the teacher asked the students to compare three triples: (a) $5\frac{3}{8}$, $5\frac{5}{8}$ and $3\frac{1}{2}$; (b) $5\frac{1}{8}$, $5\frac{1}{5}$ and $9\frac{1}{6}$; and (c) $3\frac{5}{9}$, $4\frac{1}{2}$ and $4\frac{2}{3}$. The teacher asked three students to come up the front of the board and asked each to arrange three fractions in order. Then they were asked how they did the comparison. One student made a mistake and she managed to correct the mistake herself through the teacher’s probing. The other two described the correct procedure through the teacher’s probing. The specific way of inspecting the fractions became explicit in their conversation.

EXCERPT 4

(S17 compared $5\frac{3}{8}$, $5\frac{5}{8}$ and $3\frac{1}{2}$)

T: Which part did you check first?
 S17: I first checked the integer part.

- T: The integer part first. Which is the smallest?
- S17: (pointing to $3\frac{1}{2}$, the smallest number).
- (S18 compared $5\frac{1}{8}$, $5\frac{1}{5}$ and $9\frac{1}{6}$)
- T: Which part did you check first?
- S18: I first checked the integer part.
- T: Which is the ...?
- S18: Large, 9.
- T: Yes. What follows? These two are ... You say something.
- S18: The denominators were not the same.
- T: Numerators?
- S18: Numerators are the same.
- T: Do you remember the method? Say it.
- S18: If the denominators were not the same, the smaller the denominator the larger is the fraction.

After this, the teacher showed a pre-prepared summary presentation for the purpose of a recapitulation and guided the class to go through the method of comparison again. Via questions and answers, the teacher ensured that the class knew what to compare for the different cases of fractions with the same denominators, with the same numerators, and with different denominators and numerators. After the summary, the students were asked to compare more fractions through a game in which they had more practice as well as the excitement of playing a game.

Developing basic skills via systematic guidance

Developing basic skills is essential and this can be done by various means. One essential feature is the choice of relevant learning tasks. This lesson aims to help students develop a skill of comparing mixed fractions. She did this by a deliberate sequence of the examples. In the *first pair* ($2\frac{1}{2}$ and $1\frac{1}{2}$), only the integer parts were different. The fractional parts were deliberately kept the same so that the students would need to compare the integer part only. In the *second pair* ($2\frac{3}{5}$ and $2\frac{1}{5}$), the integer parts were kept the same whereas the fractional parts were different but with the same denominators. Students were guided to compare the fractional part only and in this case they were reminded to apply what they had learned in an earlier lesson on the method of comparing proper fractions with the same denominator (i.e., when the denominators are the same, the larger the numerator the larger is the fraction).

In addition to comparing the fractions, the teacher guided the students to differentiate the mixed fractions into parts and to inspect the fractions by following a specific sequence (whole numbers, fractions, \rightarrow same denominators). Therefore, when the students came to the third pair of fractions ($3\frac{2}{3}$ and $3\frac{3}{4}$) which had the same integer part, it was very plausible for them to look for common denominators as planned by the teacher. The students were guided deliberately to give the right answer or the right step upon the teacher's request throughout the development of the basic skills. The algorithm which the teacher wanted the students to capture became very explicit in the responses by S17 and S18 in EXCERPT 4. In the sequence of consecutive examples, we notice a special way of varying a certain feature of the fractions in a systematic way in order to capture the learners' attention and let them apply some learnt skills in examples with a little variation in each case. The special arrangement of examples to a certain extent reflects how the teacher understands the particular mathematical object or skills and the arrangement thus provides an opportunity for the learners to follow the teacher's construal of the mathematical content. Such scaffolding experience is reminiscent of the concept of "teaching with variation" which has been applied in China consciously and intuitively for a long time (Gu, Huang, & Marton, 2004).

Mastering basic skills sometimes involves the learning of a useful algorithm. Take this example of comparing mixed fractions. The algorithm consists of several components: Seeing that a mixed fraction consists of an integer and a proper fraction, then compare either the integers or the proper fractions. In comparing the part containing the proper fractions, one needs to differentiate the two cases of the same denominators and different denominators. In the latter, one needs to produce equivalent fractions with new denominators. This is obviously a very complex job embedded with a lot of differentiation of various situations and subtasks. An apparent strength in the lesson is that through the systematic variation of examples and the teacher's careful guidance, it appeared that the students were working out the answers themselves based on what the teacher gave and they had already learned, thus building a competence in the method.

Opportunities for exploration / missing opportunities

In the above parts of this paper, we have analyzed how the activities in the lesson were designed to develop students' basic skills related to fractions. In this part of the paper, we present the opportunities or missing opportunities for fostering students' sense-making through teacher-student interactions. We mainly focus on the first three excerpts to provide the other side of the coin since the remaining excerpt followed the same pattern of teacher-student interaction. We take the

perspectives of the constructivists and social constructivists about learning in this part of the analysis (Cobb, 1994).

In EXCERPT 1, the teacher and students compared $2\frac{1}{2}$ with $1\frac{1}{2}$. In this comparison, students seemed to focus on the integer parts of the two fractions to determine which one is bigger. Then the teacher raised an interesting question: Should we neglect the fractional parts after comparing the integer parts of two fractions? Apparently, this question is beyond just comparing $2\frac{1}{2}$ with $1\frac{1}{2}$. It is the teachers' intention to raise this general question so that students would not just simply focus on the integer parts to compare two fractions. This question had the potential to create cognitive conflicts for students. The potential conflicts could result from the perspective one may take to seek an answer to the question. If one just compares $2\frac{1}{2}$ with $1\frac{1}{2}$, when the integer part of a fraction is greater than the integer part of another fraction, there is no need to compare the fractional part of the two fractions. Thus the answer to the question is "yes." In China, particularly, in this context, a fraction consists of integer part and proper fraction, so the answer is simply "yes". On the other hand, when one compares any two fractions which may include the case of fractions with equal integer parts, one cannot neglect the fractional parts after comparing the integer parts of two fractions. Thus, the answer is "no."

Unfortunately, the teacher missed the great opportunity to ask students to come up with answers to the question. Instead, the teacher quickly answered, "No, we need to compare the fractions" right after she raised the question. Later for the specific pair of fractions $2\frac{1}{2}$ and $1\frac{1}{2}$, the teacher said that "then we do not need to care for the fractions and we only compare the whole numbers for this pair." It is great that the teacher pointed out the potential mistake students may make when comparing two fractions, but the teacher's switching back and forth between the general aspects of comparing two fractions to the specific aspect of comparing two particular fractions may be difficult for students to follow. Such type of switching between the general aspects of comparing two fractions to the specific aspect of comparing two particular fractions is also evident in EXCERPT 2.

In EXCERPT 2, the discussion focused on the "general procedure" for comparing two mixed fractions. When S7 answer "numerator" to the teacher's inquiry about what to compare first, the teacher "marked" S7 wrong by asking: "What should we see first if we compare mixed fractions?" Then, S7 changed the answer to the integers. The teacher acknowledged the correct response from S7, and then probed

for the next step in the comparison of two fractions. Apparently, the teacher did not accept S7's response of "compare fractions." Again, S7 changed his/her answer and the teacher accepted the response. This dialogue can hardly be called discussion. It is like students trying to guess answers the teacher expected. The "correct" answers the teacher expected were imposed onto students. In addition, the teacher did not give any explanation why one should compare integers first and then compare denominators of the fractional parts. The two responses from S7 which were marked incorrect by the teacher are actually correct responses for comparing $2\frac{3}{5}$ and $2\frac{1}{5}$. The response provided by S8 (the numerators) is correct as well. From the EXCERPT 2, we can clearly see that students rely on the teacher as the only authority. Even students provide sensible responses, the teacher paid little attention to them but her scripted agenda for coming up with the general procedure of comparing two mixed fractions. Since in the previous lessons, those general rules and procedures have been learnt, what the teacher tried to solicit is obviously the application of those knowledge in these particular situation or examples. It may be common and helpful for consolidating basic skills but there is a trade-off for such practice.

As we pointed out before, the teacher's style of questions belongs to a style which led the class to follow a specific path closely. The students were led to inspect different parts of the fractions in a specific sequence (whole numbers, fractions, denominators) and they were not encouraged to skip any part. The advantage of this type of teaching is to provide students with the general procedure of comparing two fractions. On the other hand, this type of structured teaching is not true classroom discourse even though superficially there were a number of teacher-student dialogues. This is the typical pattern of "Funnel" by Wood (1998). From a long-term perspective, this type of teaching may limit students' flexibility of thinking. For example, the teacher instructed students that they *have to* compare integer parts first when they compare two mixed fractions. However, in many cases of fractions with the same integer parts, they actually compare fractional parts quickly and directly to arrive at correct answers. The teacher also instructed that students *have to* compare denominator first when comparing the fractional parts. However, in many cases students can compare the fractions as a whole for correct answers (e.g., $\frac{1}{2}$ and $\frac{1}{4}$, referring to this comparison, different strategies could be developed when comparing two proper fractions in advance. For example, based on the meaning of fraction, when the numerator is the same, the larger the denominator, the smaller the fraction). Keeping the general procedure in mind, the students should apply these

procedures flexibly in different contexts. Therefore, it is important to differentiate the goal of training a routine algorithm from the goal of developing an awareness of the various strategies of comparing fractions in a variety of cases.

Through comparing $3\frac{2}{3}$ and $3\frac{3}{4}$, the lesson continued to develop the presentation of general procedures of comparing two fractions. Again, there are a number of student-teacher interactions. However, this part of the interaction seemed to be much smoother than those in the previous two excerpts because students' responses matched what the teacher expected. It is interesting to see the blank-filling transparency shown in Figure 1. This is a well-structured and pre-prepared activity. This activity is certainly a critical step in comparing two fractions. However, is it possible that since the activity is too well-structured it took away all of the challenges necessary in learning mathematics? For example, it may be improved if the teacher provides students with the following problems: $3\frac{2}{3}$ and $3\frac{2}{5}$, and requests non-routine strategies to solve it. However, it may be that the teacher paid too much attention to helping students master the standards procedures, hence, she neglected multiple alternatives.

Conclusions and Recommendations

In this paper, we have analyzed a well-structured lesson to examine the opportunities and missing opportunities for students' learning. The teacher structured the sequential tasks to help students learn the procedures of comparing fractions. The follow-up interviews with students and teachers clearly showed that students mastered the procedures well. However, the analysis of the lesson showed that students also missed many opportunities for independent exploration. Well-structured is a common feature of the lessons where Chinese learn mathematics (Fan, Wong, Cai, & Li, 2004). The teacher has planned and delivered the lesson in such a way that students had little room to think independently. Instead, students mainly followed the teacher's "planned frame" to learn what was prior determined by the teacher.

From the follow-up interview with the teacher, we learned that the teacher was well aware of the missing opportunities. While she tried real hard to engage students, it was not easy to "let it go." In addition to engaging students in good problem-solving activities, the type of engagement is vitally important. The type of engagement she created is less ideal for fostering students' higher-order thinking skills. The analysis of the lesson demonstrated the strengths of a well-structured lesson in enhancing the learning of basic skills and the potential limitations in fostering higher-order thinking skills. For example, there are alternative ways of handling students'

responses and posing problems as suggested in the analysis of EXCERPTS 1 and 2. We believe that both basic and higher-order thinking skills are important. We also believe that it is not necessary to sacrifice basic skills for higher-order thinking, nor higher-order thinking for basic skills. Although according to various learning theories, there may be different procedures to develop basic skills and higher order thinking skills, we believe that they can develop together. In this paper, we present a case study to help us reflect upon the missing opportunities for higher order thinking skills in a lesson targeting for basic skills. Learning opportunities depend on the nature of the tasks and the teacher's choice in orchestrating the classroom discourse. The challenge we face with is how we should design instruction to develop both (Brownell, 1947; Davis, 1992; Hiebert & Carpenter, 1992). Apparently, more research is needed to exploring the kinds of instructional design that can develop both basic skills and higher-order thinking.

At the end of this paper, we provide a recommendation of changing classroom interactions for well-structured lessons for the purpose of increasing students' learning opportunities. The instructional activities in the lesson that show a very systematic choice of variation and clear focus, may serve well the goal of teaching a specific basic skill. However, the opportunity for learning depends significantly on the nature of interaction generated in the classroom discourse. To be effective in the classroom, students have to have more opportunity to express their ideas and justify their answers (Hiebert & Wearne, 1993; Lampert, 1990). There are a number of factors that may influence the nature of classroom interactions. One of the predominant factors is the amount of time allocated to solving problems and discussing solution efforts (Henningsen & Stien, 1997; Michelle, VanderStoep, & Yu, 1993; Stiger & Hiebert, 1999). In classrooms with sound discourse, the discussion of a problem and its alternative solutions usually takes longer than the demonstration of a routine classroom activity. Hiebert and Wearne found that classrooms with a primary focus on understanding used fewer problems and spent more time on each of them, compared to those classrooms without a primary focus on understanding (Hiebert & Wearne, 1993). Moreover, in classrooms, teachers ask more conceptually-oriented questions (e.g., describe a strategy or explain underlying reasoning for getting an answer) and fewer recall questions than teachers in the classrooms without a primary focus on problem solving. The findings are consistent with what has been found in cross-cultural comparative studies (e.g., Michelle, VanderStoep, & Yu, 1993; Stiger & Hiebert, 1999).

The teachers' role in guiding mathematical discourse is a highly complex activity. Besides devoting an appropriate amount of time to the discussion of problems, "teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with

varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge” (NCTM, 2000). In other words, it is important for teachers to provide enough support for students’ mathematical exploration, but not so much support that they take over the process of thinking for their students (e.g., Ball, 1993; Lampert, 1990; Hiebert et al., 1997).

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